

# PARALLEL CONSTRUCTION OF SIMULTANEOUS DETERMINISTIC FINITE AUTOMATA ON SHARED- MEMORY MULTICORES

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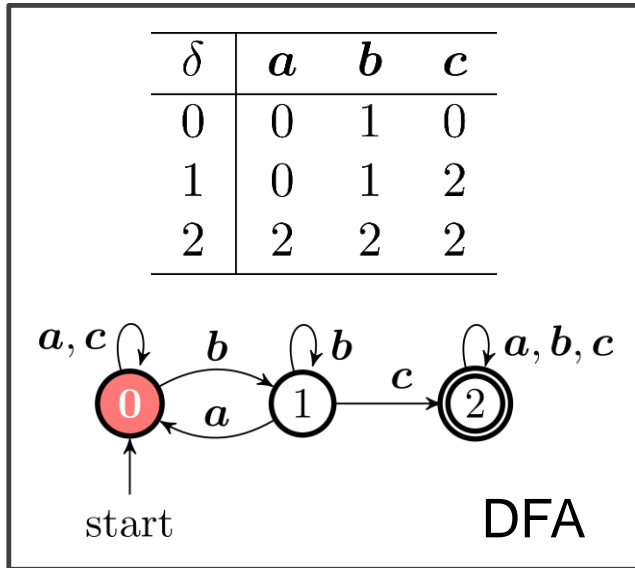
46<sup>th</sup> International Conference of Parallel Processing  
Bristol, United Kingdom in August 14 - 17, 2017

# Motivation

- String pattern matching with finite automata (FAs) is a well-established method across many areas.
  - ▣ Text editors
  - ▣ Compiler front-ends
  - ▣ Internet search engines
  - ▣ Security and DNA sequence analysis
- The sequential FA algorithm has linear complexity in the size of the input.
  - ▣ Significant research effort has been spent on parallelizing FA matching to improve the sequential performance
  - ▣ → Hard to be parallelized due to the dependency between state transitions

# Motivation (cont.)

## □ Limitation of parallel FA matching



Input:

*bbacaaabcc*

9 steps

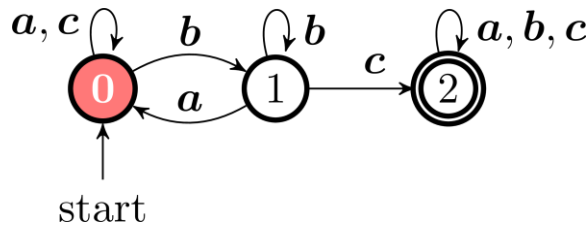
start **0**  $\rightarrow$   $p_0$ : 

1	1	0	0	0	0	1	2	2
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# Motivation (cont.)

## □ Limitation of parallel FA matching

$\delta$	$a$	$b$	$c$
0	0	1	0
1	0	1	2
2	2	2	2



Input: *bbacaaabcc* 9 steps

start 0  $\rightarrow$   $p_0$ : 110000122

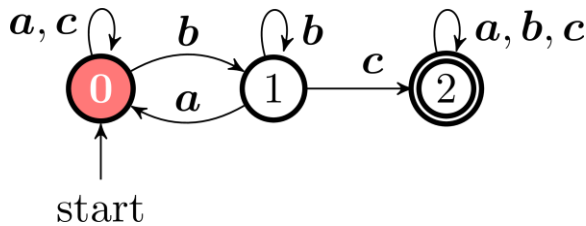
$\downarrow \mathcal{O}\left(\frac{n}{|P|}\right)$

chunks:  $c_0$  *bb a*  $c_1$  *c a a*  $c_2$  *b c c*  
 3 steps      3 steps      3 steps

# Motivation (cont.)

## □ Limitation of parallel FA matching

$\delta$	$a$	$b$	$c$
0	0	1	0
1	0	1	2
2	2	2	2



Input: b b a c a a b c c 9 steps

start 0  $\rightarrow p_0$ : 1 1 0 0 0 0 1 2 2

$\downarrow \mathcal{O}\left(\frac{n}{|P|}\right)$

chunks:  $c_0$  b b a  $c_1$  c a a  $c_2$  b c c

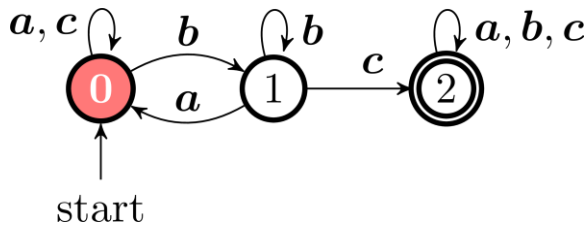
start 0  $\rightarrow p_0$ : 1 1 0

What is the start state?

# Motivation (cont.)

## □ Limitation of parallel FA matching

$\delta$	$a$	$b$	$c$
0	0	1	0
1	0	1	2
2	2	2	2



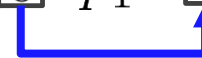
Input: *b b a c a a b c c* 9 steps

start 0  $\rightarrow$   $p_0$ : 1 1 0 0 0 0 1 2 2

$\downarrow \mathcal{O}\left(\frac{n}{|P|}\right)$

chunks:  $c_0$  *b b a*  $c_1$  *c a a*  $c_2$  *b c c*

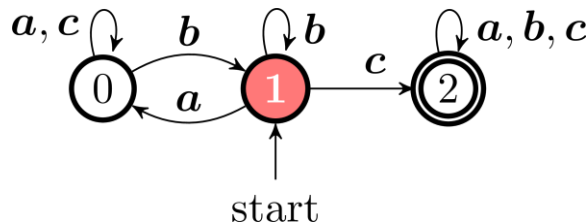
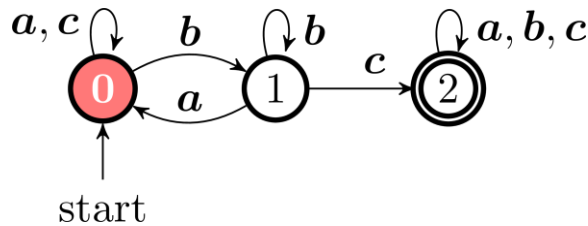
start 0  $\rightarrow$   $p_0$ : 1 1 0  $p_1$ : 0 0 0



# Motivation (cont.)

## □ Limitation of parallel FA matching

$\delta$	<i>a</i>	<i>b</i>	<i>c</i>
0	0	1	0
1	0	1	2
2	2	2	2



Input: *b b a c a a b c c* 9 steps

start **0**  $\rightarrow$   $p_0$ : 1 1 0 0 0 0 1 2 2

$\downarrow \mathcal{O}\left(\frac{n}{|P|}\right)$

chunks:  $c_0$  *b b a*  $c_1$  *c a a*  $c_2$  *b c c*

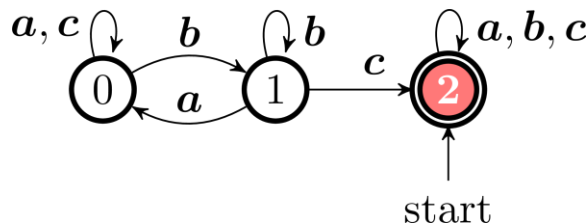
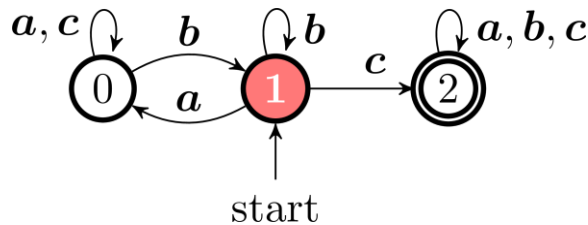
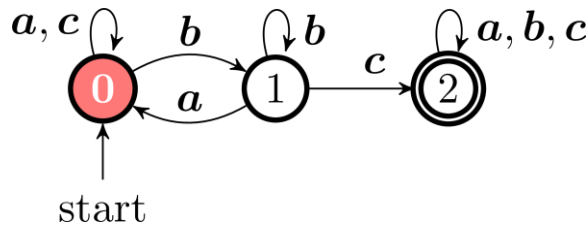
start **0**  $\rightarrow$   $p_0$ : 1 1 0  $p_1$ : 0 0 0

start **1**  $\rightarrow$  2 2 2

# Motivation (cont.)

## □ Limitation of parallel FA matching

$\delta$	<i>a</i>	<i>b</i>	<i>c</i>
0	0	1	0
1	0	1	2
2	2	2	2



Input: *b b a c a a b c c* 9 steps

start **0**  $\rightarrow$   $p_0$ : 1 1 0 0 0 0 1 2 2

$\downarrow \mathcal{O}\left(\frac{n}{|P|}\right)$

chunks:  $c_0$  *b b a*  $c_1$  *c a a*  $c_2$  *b c c*

start **0**  $\rightarrow$   $p_0$ : 1 1 0  $p_1$ : 0 0 0

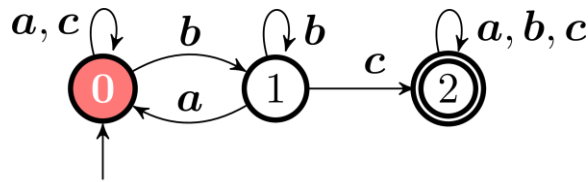
start **1**  $\rightarrow$  2 2 2

start **2**  $\rightarrow$  2 2 2

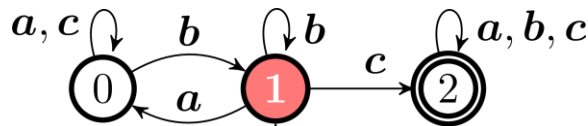
# Motivation (cont.)

## □ Limitation of parallel FA matching

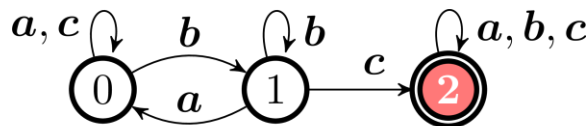
$\delta$	$a$	$b$	$c$
0	0	1	0
1	0	1	2
2	2	2	2



start



start



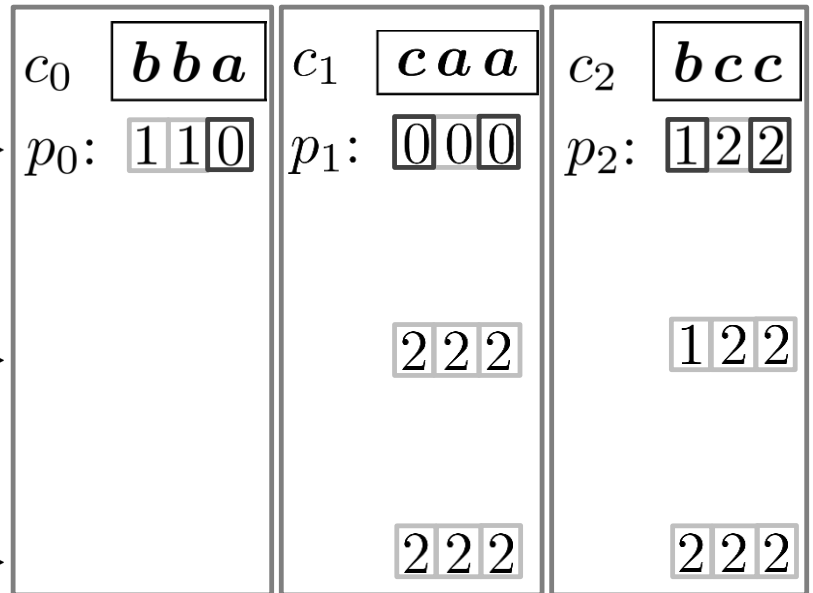
start

Input: b b a c a a b c c 9 steps

start **0**  $\rightarrow$   $p_0$ : 1 1 0 0 0 0 1 2 2

$\downarrow$   ~~$\mathcal{O}\left(\frac{n}{|P|}\right)$~~   $\mathcal{O}\left(\frac{n \times |Q|}{|P|}\right)$

chunks:



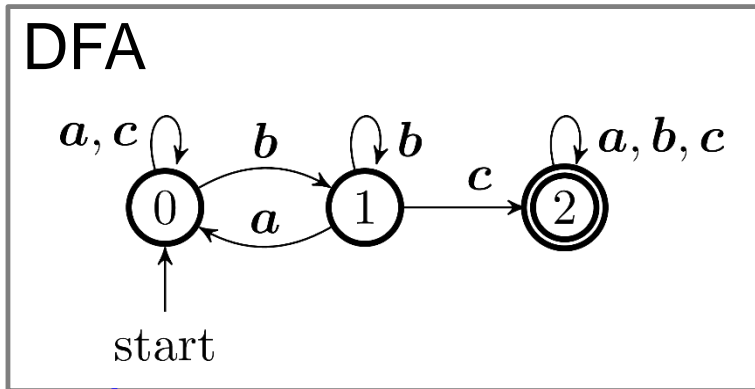
3 steps

$3 \times 3$  steps

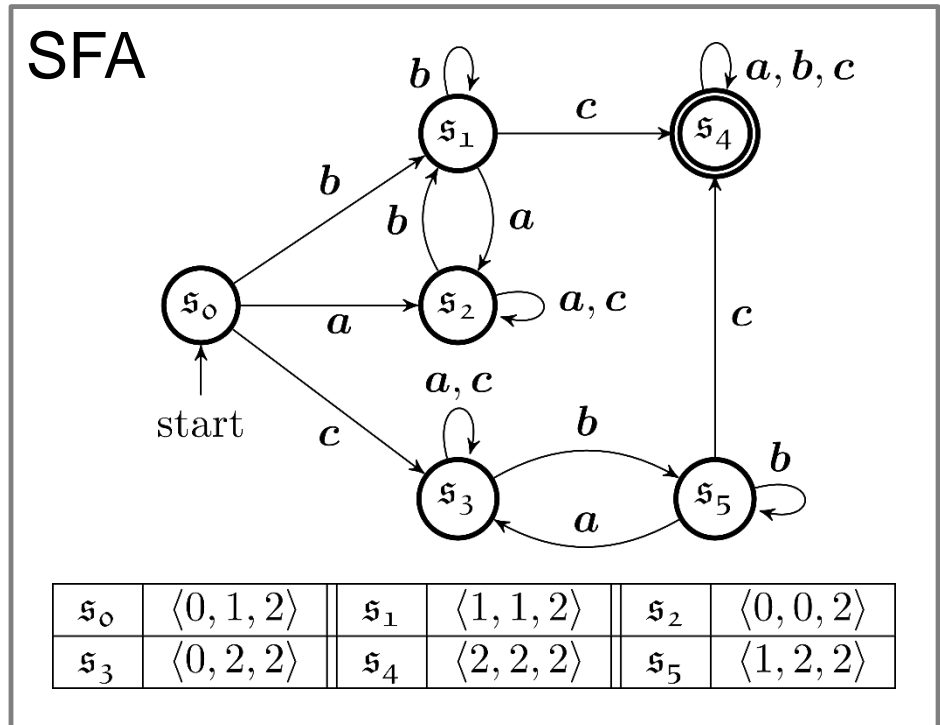
$3 \times 3$  steps

# Motivation (cont.)

- Simultaneous Finite Automata (SFAs)
  - ▣ Accumulated state transition information
  - ▣ Simulates the parallel execution of  $|Q|$  DFAs on a single DFA

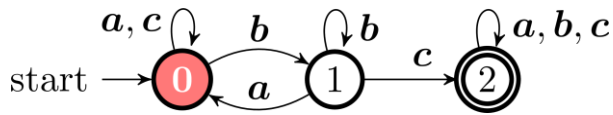


**SFA construction**

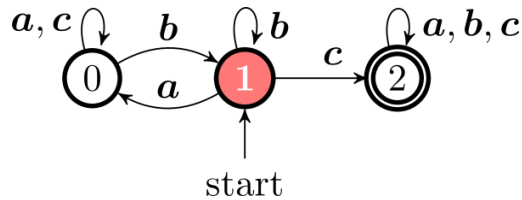


# Motivation (cont.)

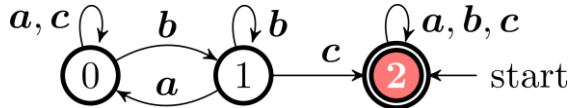
## □ Parallel FA matching



chunks:  $c_0$  ***bba***  $c_1$  ***caa***  $c_2$  ***bcc***  
 start **0** →

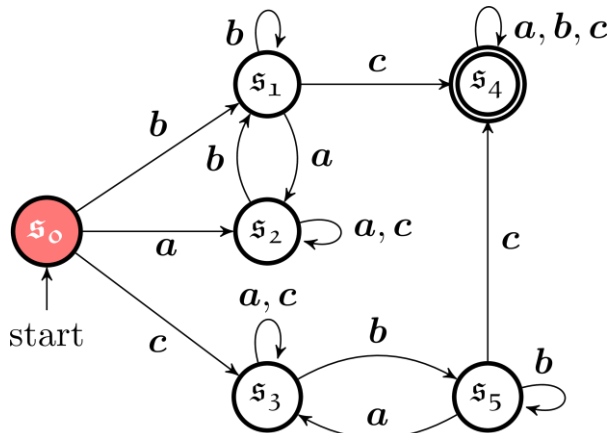


start **1** →



start **2** →

## □ Parallel SFA matching

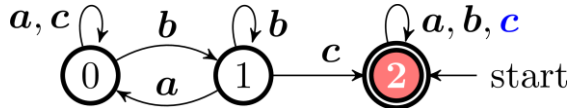
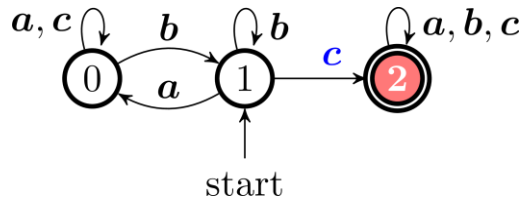
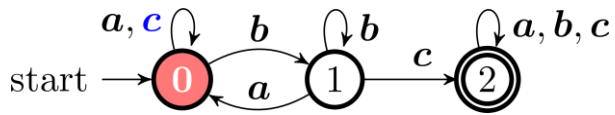


$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$	$s_4$	$\langle 2, 2, 2 \rangle$	$s_5$	$\langle 1, 2, 2 \rangle$

chunks:  $c_0$  ***bba***  $c_1$  ***caa***  $c_2$  ***bcc***  
 start  **$s_0$**  →

# Motivation (cont.)

## □ Parallel FA matching

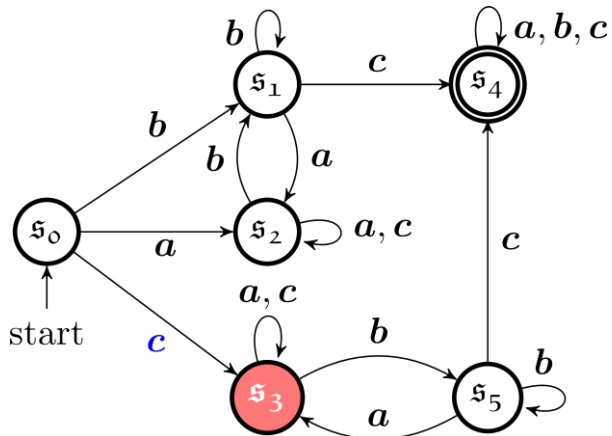


chunks:  $c_0$  **bb a**  $c_1$  **c a a**  $c_2$  **b c c**  
 start 0 → **0**

start 1 → **2**

start 2 → **2**

## □ Parallel SFA matching

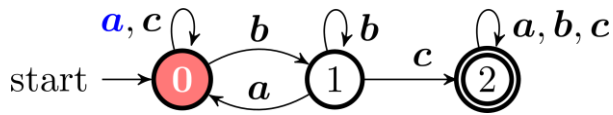


$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$	$s_4$	$\langle 2, 2, 2 \rangle$	$s_5$	$\langle 1, 2, 2 \rangle$

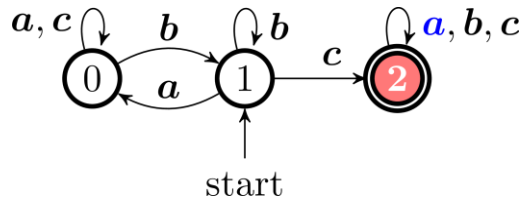
chunks:  $c_0$  **bb a**  $c_1$  **c a a**  $c_2$  **b c c**  
 start  $s_0$  →  **$s_3$**

# Motivation (cont.)

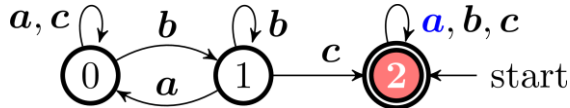
## □ Parallel FA matching



chunks:  $c_0$  **b b a**  $c_1$  **c a a**  $c_2$  **b c c**  
 start 0 → 0 **0**

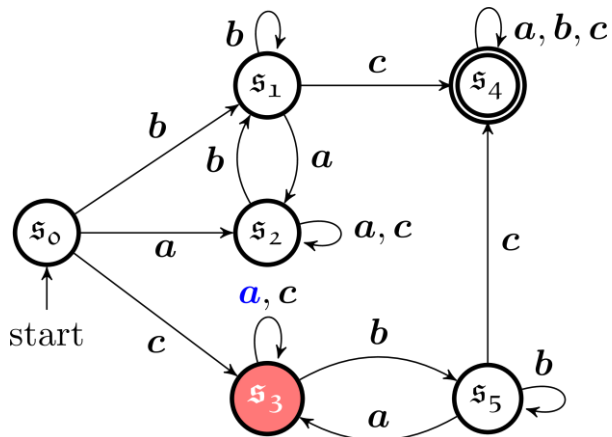


start 1 → 2 **2**



start 2 → 2 **2**

## □ Parallel SFA matching

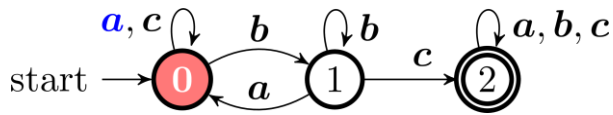


$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$	$s_4$	$\langle 2, 2, 2 \rangle$	$s_5$	$\langle 1, 2, 2 \rangle$

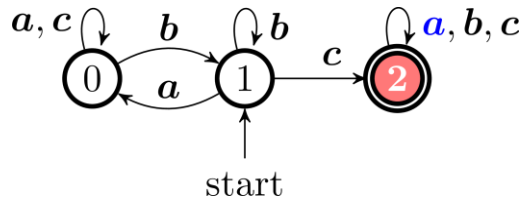
chunks:  $c_0$  **b b a**  $c_1$  **c a a**  $c_2$  **b c c**  
 start  $s_0$  →  $s_3$   **$s_3$**

# Motivation (cont.)

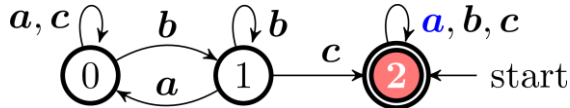
## □ Parallel FA matching



chunks:  $c_0$  **b b a**  $c_1$  **c a** a  $c_2$  **b c c**  
 start 0 → 0 0 0

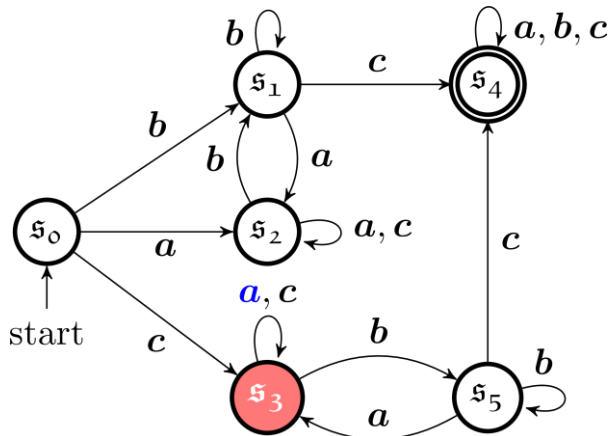


start 1 → 2 2 2



start 2 → 2 2 2

## □ Parallel SFA matching

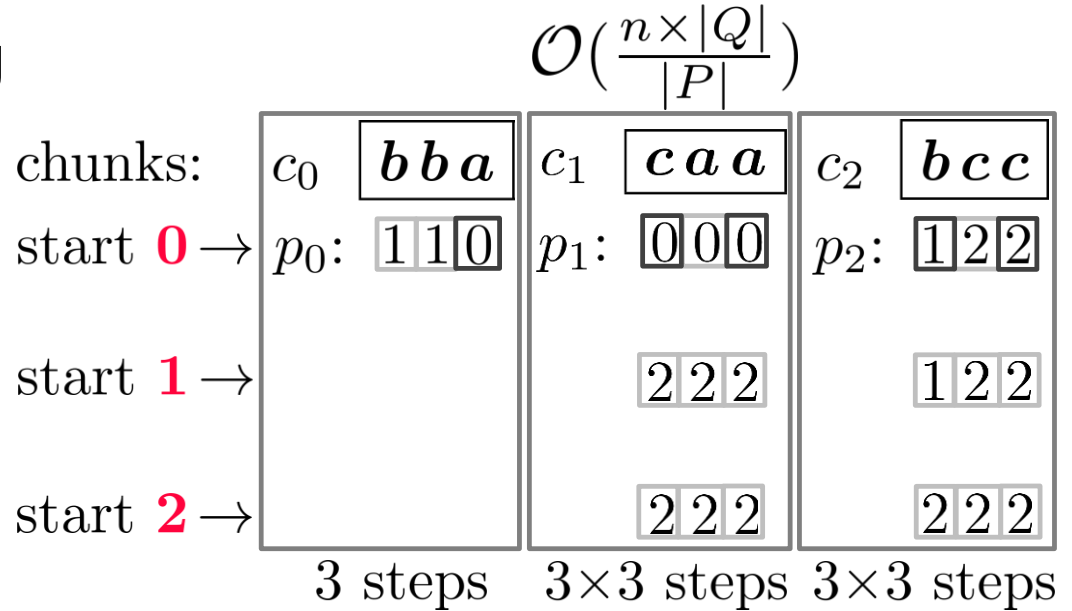
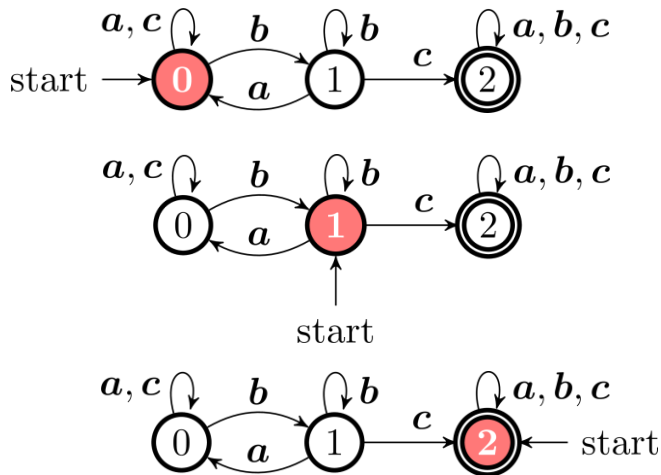


$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
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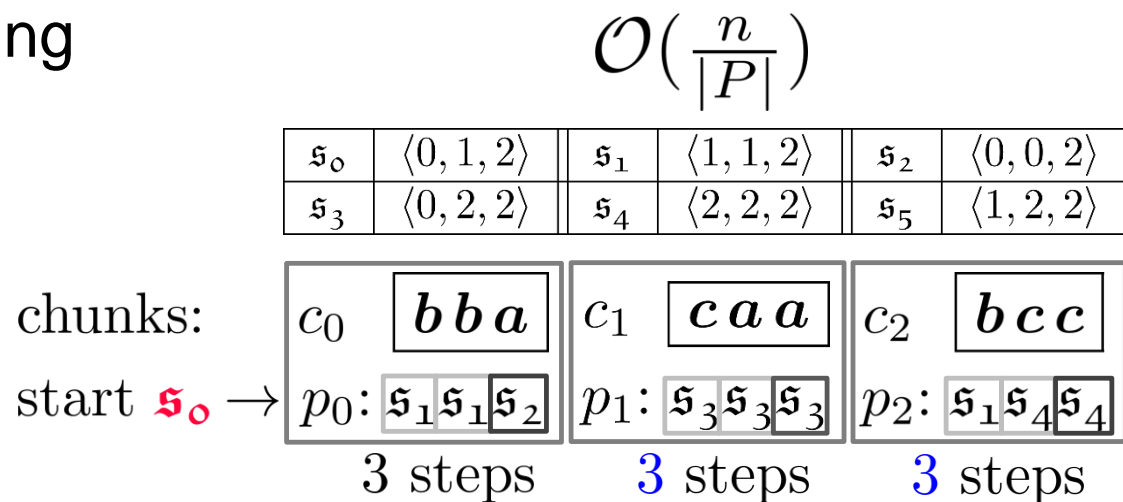
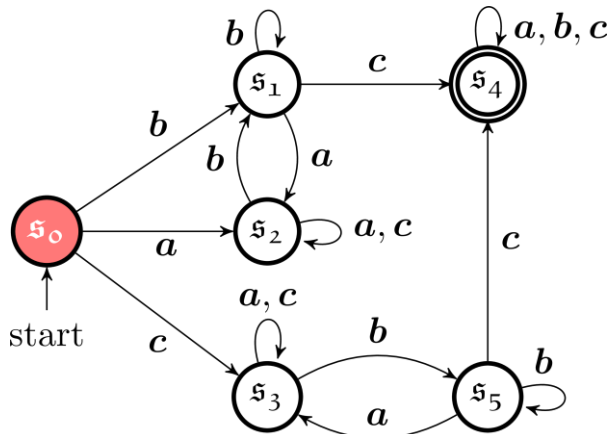
chunks:  $c_0$  **b b a**  $c_1$  **c a** a  $c_2$  **b c c**  
 start  $s_0$  →  $s_3 s_3$   $s_3$

# Motivation (cont.)

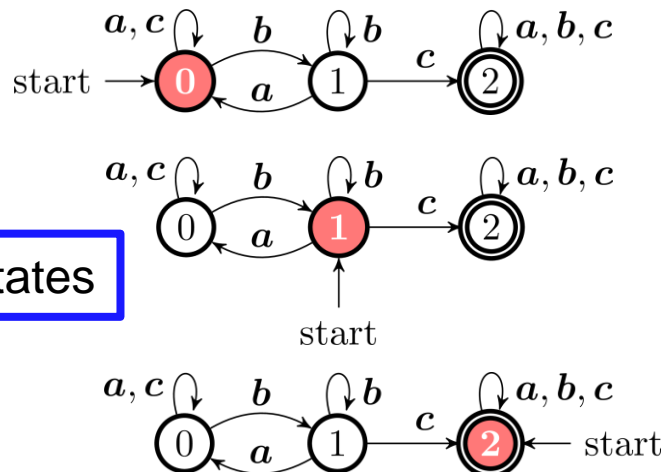
## Parallel FA matching



## Parallel SFA matching



# Motivation (cont.)



3 states

chunks:

start **0** →

start **1** →

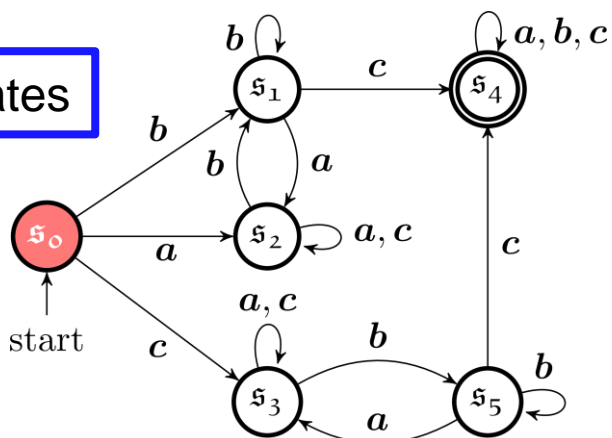
start **2** →

$c_0$	$\boxed{bba}$	$c_1$	$\boxed{caa}$	$c_2$	$\boxed{bcc}$
start <b>0</b> →	$p_0: \boxed{110}$	$p_1: \boxed{000}$		$p_2: \boxed{122}$	
start <b>1</b> →		$\boxed{222}$		$\boxed{122}$	
start <b>2</b> →		$\boxed{222}$		$\boxed{222}$	
	3 steps	3×3 steps		3×3 steps	

**Problem** of SFA construction:

SFA size is exponential in number of FA-states  $\mathcal{O}(|Q|^{|Q|})$

6 states



chunks:

start  **$s_0$**  →

$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$	$s_4$	$\langle 2, 2, 2 \rangle$	$s_5$	$\langle 1, 2, 2 \rangle$
$c_0$	$\boxed{bba}$	$c_1$	$\boxed{caa}$	$c_2$	$\boxed{bcc}$
start <b><math>s_0</math></b> →	$p_0: \boxed{s_1 s_1 s_2}$	$p_1: \boxed{s_3 s_3 s_3}$		$p_2: \boxed{s_1 s_4 s_4}$	
	3 steps	3 steps		3 steps	

# Our contributions

1. Introduce **fingerprint-based hashing** of SFA-states to speed up state comparisons.
2. Provide **x86 SIMD-based transposition kernels** for SFA-state construction to leverage data-parallelism and cache-locality.
3. Perform **in-memory compression** of SFA-states to mitigate the space constraints of large problems.
4. **Parallelize** SFA construction for shared-memory multicores with lock-free synchronization on all data-structures including **thread-local queues** supporting work-stealing.

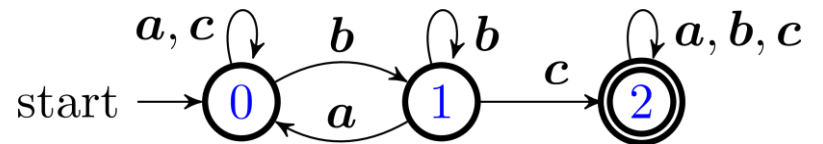
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Start with the initial state  $s_I$ .

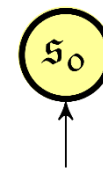


DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{\}$

$Q_{tmp} = \{s_I\}$

$s_o$	$\langle 0, 1, 2 \rangle$
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start  
SFA

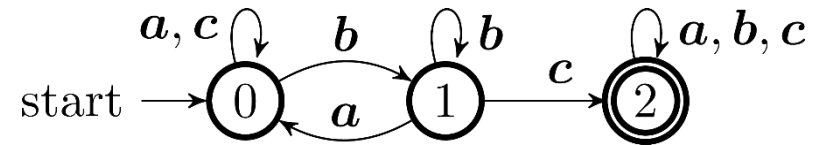
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10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Until no more states to process



DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{\}$

$Q_{tmp} = \{s_o\}$

$s_o$	$\langle 0, 1, 2 \rangle$
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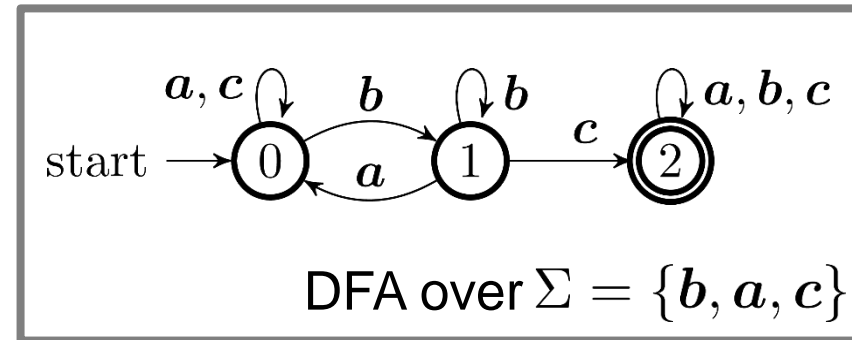
start  
SFA

# Sequential SFA construction

```

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11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```



$Q_s = \{\}$

$Q_{tmp} = \{s_o\}$

$s = s_o$

$s_o$	$\langle 0, 1, 2 \rangle$
-------	---------------------------



start  
SFA

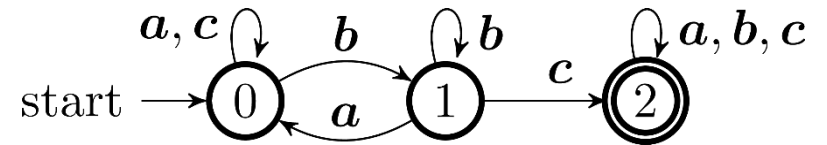
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```

Insert  $s$  into the processed set



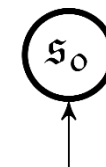
DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_o\}$

$Q_{tmp} = \{\}$

$s = s_o$

$s_o$	$\langle 0, 1, 2 \rangle$
-------	---------------------------



start  
SFA

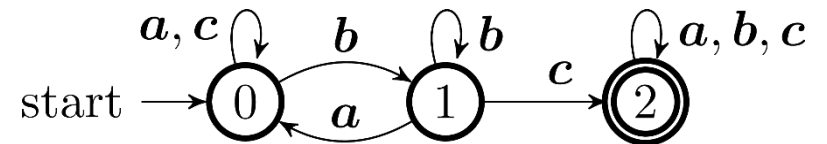
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Iterate with every symbol



DFA over  $\Sigma = \{b, a, c\}$

$$Q_s = \{s_o\}$$

$$Q_{tmp} = \{\}$$

$$s = s_o$$

$s_o$	$\langle 0, 1, 2 \rangle$
-------	---------------------------



start  
SFA

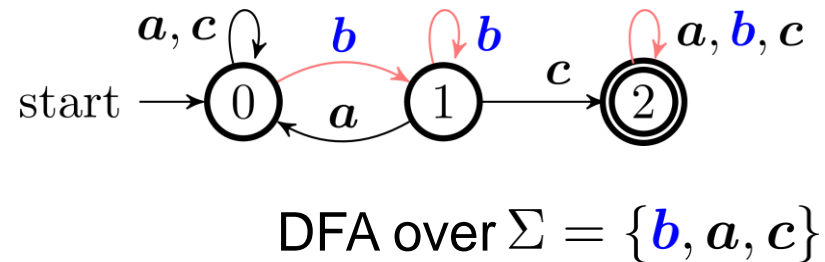
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Find new states



$$Q_s = \{s_o\}$$

$$Q_{tmp} = \{\}$$

$$s_o = \langle 0, 1, 2 \rangle$$

$$s_o = \{0, 1, 2\}$$

↓ ↓ ↓  $b$

$$s_{next} = \{1, 1, 2\}$$



start  
SFA

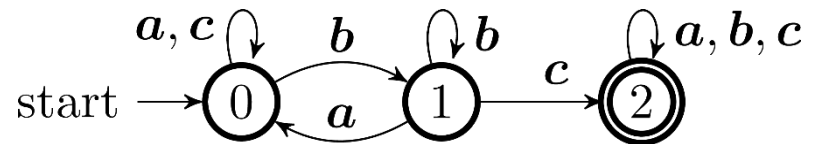
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Update the SFA transition function



DFA over  $\Sigma = \{b, a, c\}$

$$Q_s = \{s_0\}$$

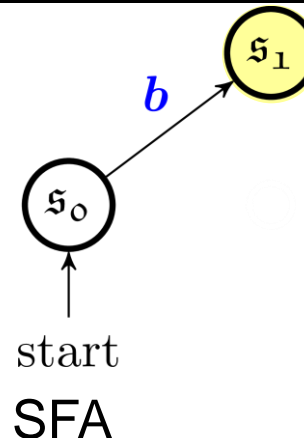
$$Q_{tmp} = \{\}$$

$$s_0 = \{0, 1, 2\}$$

$\downarrow \downarrow \downarrow b$

$$s_1 = \{1, 1, 2\}$$

$s_0$	$\langle 0, 1, 2 \rangle$
$s_1$	$\langle 1, 1, 2 \rangle$

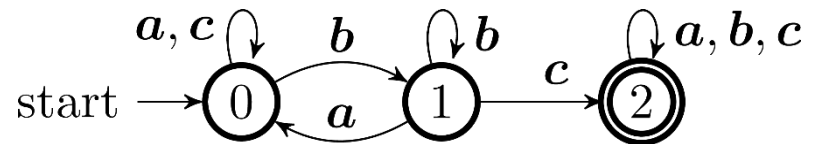


# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10   $I_s \leftarrow \{s_I\}$ 
11   $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 
  
```

Check existence &  
add new state to the set  
(set membership test)



DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_0\}$

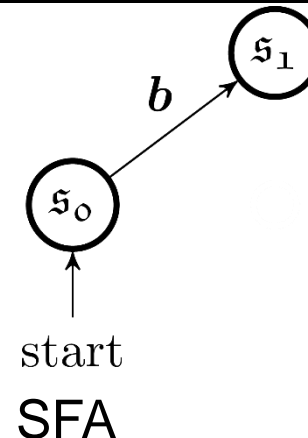
$Q_{tmp} = \{s_1\}$

$s_0 = \{0, 1, 2\}$

$\downarrow \downarrow \downarrow b$

$s_1 = \{1, 1, 2\}$

$s_0$	$\langle 0, 1, 2 \rangle$
$s_1$	$\langle 1, 1, 2 \rangle$



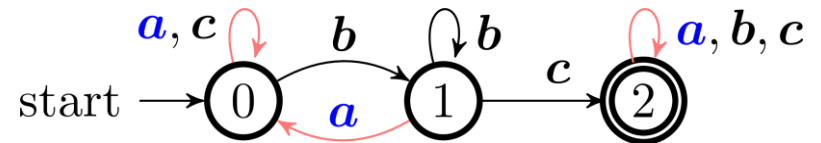
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10   $I_s \leftarrow \{s_I\}$ 
11   $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

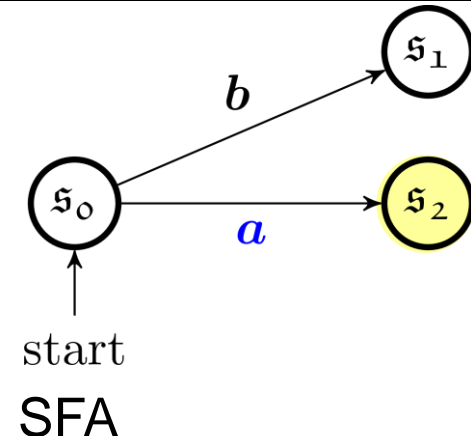
Generate a next state with symbol  $a$



DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_0\}$   
 $Q_{tmp} = \{s_1, s_2\}$   
 $s_0 = \{0, 1, 2\}$   
 $\downarrow \downarrow \downarrow a$   
 $s_2 = \{0, 0, 2\}$

$s_0$	$\langle 0, 1, 2 \rangle$
$s_1$	$\langle 1, 1, 2 \rangle$
$s_2$	$\langle 0, 0, 2 \rangle$



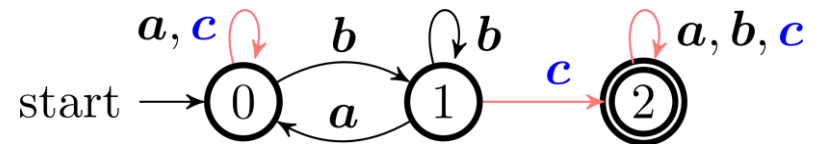
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Generate a next state with symbol  $c$



DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_0\}$

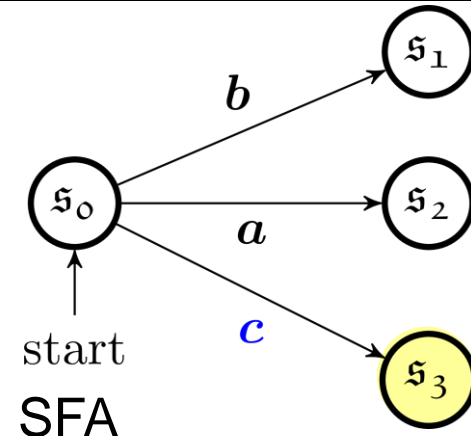
$Q_{tmp} = \{s_1, s_2, s_3\}$

$s_0 = \{0, 1, 2\}$

↓ ↓ ↓  $c$

$s_3 = \{0, 2, 2\}$

$s_0$	$\langle 0, 1, 2 \rangle$
$s_1$	$\langle 1, 1, 2 \rangle$
$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$



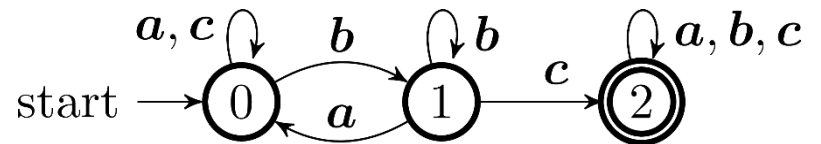
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

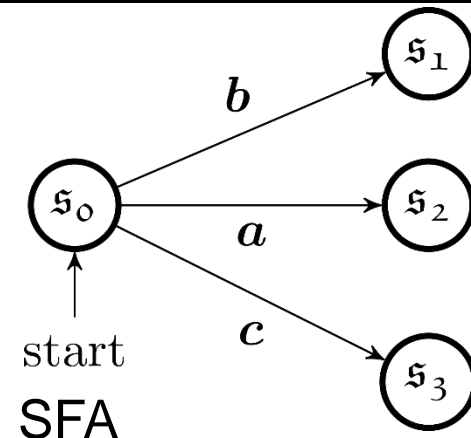
Choose the unprocessed state  $s_1$



DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_0, s_1\}$   
 $Q_{tmp} = \{s_2, s_3\}$   
 $s = s_1$

$s_0$	$\langle 0, 1, 2 \rangle$
$s_1$	$\langle 1, 1, 2 \rangle$
$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$



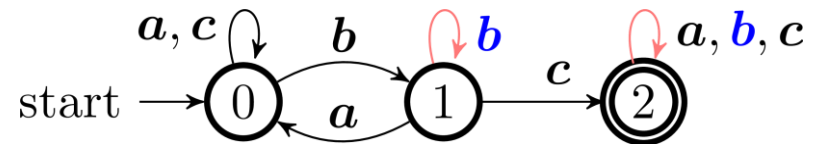
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10   $I_s \leftarrow \{s_I\}$ 
11   $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Generate a next state with symbol  $b$



DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_0, s_1\}$

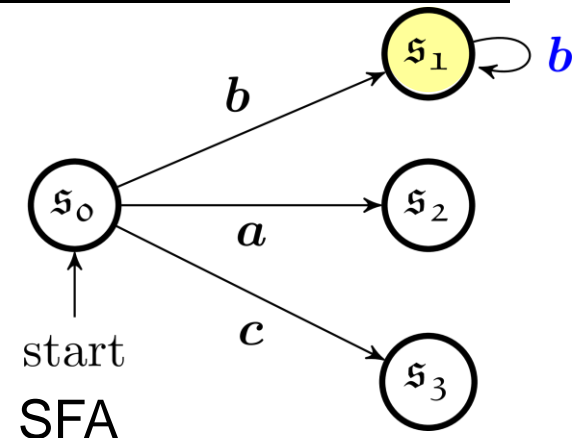
$Q_{tmp} = \{s_2, s_3\}$

$s_1 = \{1, 1, 2\}$

↓ ↓ ↓  $b$

$s_{next} = \{1, 1, 2\}$

$s_0$	$\langle 0, 1, 2 \rangle$
$s_1$	$\langle 1, 1, 2 \rangle$
$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$



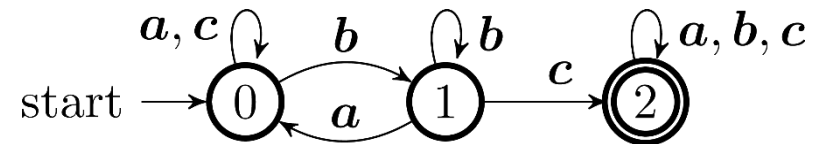
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

Until no more states to process

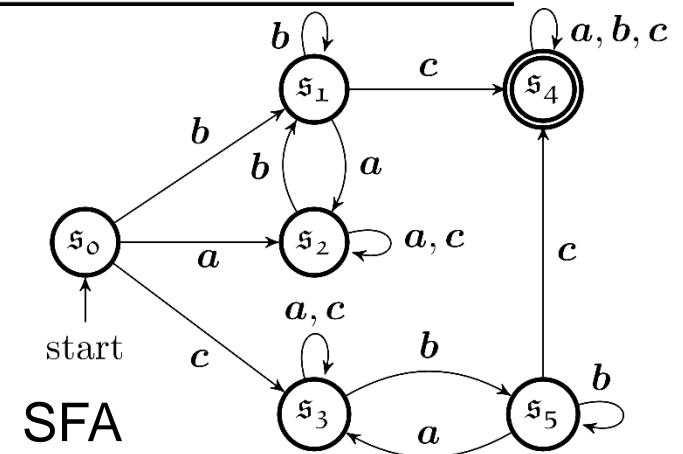


DFA over  $\Sigma = \{b, a, c\}$

$Q_s = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$

$Q_{tmp} = \{\}$

$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$	$s_4$	$\langle 2, 2, 2 \rangle$	$s_5$	$\langle 1, 2, 2 \rangle$



SFA

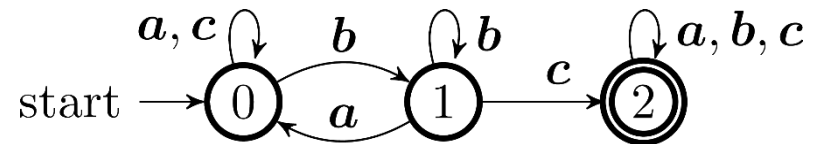
# Sequential SFA construction

```

1  $Q_s \leftarrow \emptyset, Q_{tmp} \leftarrow \{s_I\}$ 
2 while  $Q_{tmp} \neq \emptyset$  do
3   choose and remove a SFA state  $s$  from  $Q_{tmp}$ 
4    $Q_s \leftarrow Q_s \cup \{s\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad s_{next}(q) := \bigcup_{q' \in s(q)} \delta(q', \sigma)$ 
7      $\delta_s[s, \sigma] \leftarrow s_{next}$ 
8     if  $s_{next} \notin Q_s, Q_{tmp}$  then
9        $Q_{tmp} \leftarrow Q_{tmp} \cup \{s_{next}\}$ 

```

Set the initial and the final state



DFA over  $\Sigma = \{b, a, c\}$

```

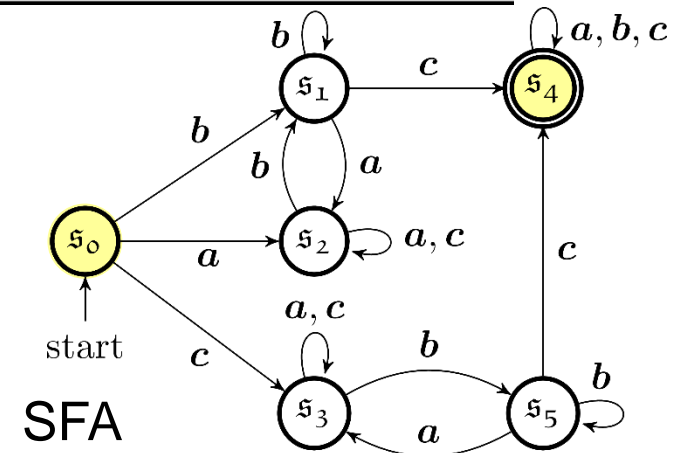
10  $I_s \leftarrow \{s_I\}$ 
11  $F_s \leftarrow \{s \in Q_s \mid \exists q \in I \mid s(q) \cap F \neq \emptyset\}$ 

```

$Q_s = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$

$Q_{tmp} = \{\}$

$s_0$	$\langle 0, 1, 2 \rangle$	$s_1$	$\langle 1, 1, 2 \rangle$	$s_2$	$\langle 0, 0, 2 \rangle$
$s_3$	$\langle 0, 2, 2 \rangle$	$s_4$	$\langle 2, 2, 2 \rangle$	$s_5$	$\langle 1, 2, 2 \rangle$



SFA

# Optimizing SFA construction

```
1  $Q_s \leftarrow \emptyset, Q_{\text{tmp}} \leftarrow \{\mathfrak{s}_I\}$ 
2 while  $Q_{\text{tmp}} \neq \emptyset$  do
3   choose and remove a SFA state  $\mathfrak{s}$  from  $Q_{\text{tmp}}$ 
4    $Q_s \leftarrow Q_s \cup \{\mathfrak{s}\}$ 
5   forall the  $\sigma \in \Sigma$  do
6      $q \in Q \quad \mathfrak{s}_{\text{next}}(q) := \bigcup_{q' \in \mathfrak{s}(q)} \delta(q', \sigma)$ 
7      $\delta_s[\mathfrak{s}, \sigma] \leftarrow \mathfrak{s}_{\text{next}}$ 
8     if  $\mathfrak{s}_{\text{next}} \notin Q_s, Q_{\text{tmp}}$  then
9        $Q_{\text{tmp}} \leftarrow Q_{\text{tmp}} \cup \{\mathfrak{s}_{\text{next}}\}$ 
10  $I_s \leftarrow \{\mathfrak{s}_I\}$ 
11  $F_s \leftarrow \{\mathfrak{s} \in Q_s \mid \exists q \in I \mid \mathfrak{s}(q) \cap F \neq \emptyset\}$ 
```

$$\mathcal{O}\left(\sum_{i=1}^{|Q_s|} \sum_{j=1}^{|\Sigma|} (|\mathcal{Q}| + |\mathcal{Q}| \times i)\right) = \mathcal{O}\left(\frac{1}{2} \times |\Sigma| \times |Q| \times \underline{|Q_s|} \times (\underline{|Q_s|} + 3)\right)$$

An SFA size  $|Q_s|$  is  $\underline{\mathcal{O}(|Q|^{|Q|})}$  in the worst case

**Exponential state-growth**

# Optimizing SFA construction

---

```

1   $Q_s \leftarrow \emptyset, Q_{\text{tmp}} \leftarrow \{\mathfrak{s}_I\}$ 
2  while  $Q_{\text{tmp}} \neq \emptyset$  do
3      choose and remove a SFA state  $\mathfrak{s}$  from  $Q_{\text{tmp}}$ 
4       $Q_s \leftarrow Q_s \cup \{\mathfrak{s}\}$ 
5      forall the  $\sigma \in \Sigma$  do
6           $q \in Q \quad \mathfrak{s}_{\text{next}}(q) := \bigcup_{q' \in \mathfrak{s}(q)} \delta(q', \sigma) \leftarrow$  Parameterized transposition
7           $\delta_s[\mathfrak{s}, \sigma] \leftarrow \mathfrak{s}_{\text{next}}$ 
8          if  $\mathfrak{s}_{\text{next}} \notin Q_s, Q_{\text{tmp}}$  then  $\leftarrow$  Fingerprint-based hashing
9               $Q_{\text{tmp}} \leftarrow Q_{\text{tmp}} \cup \{\mathfrak{s}_{\text{next}}\}$ 
10  $I_s \leftarrow \{\mathfrak{s}_I\}$ 
11  $F_s \leftarrow \{\mathfrak{s} \in Q_s \mid \exists q \in I \mid \mathfrak{s}(q) \cap F \neq \emptyset\}$ 

```

---

$$\mathcal{O}\left(\sum_{i=1}^{|Q_s|} \sum_{j=1}^{|\Sigma|} (|\mathcal{Q}| + |\mathcal{Q}| \times i)\right) = \mathcal{O}\left(\frac{1}{2} \times |\Sigma| \times |Q| \times \underline{|Q_s|} \times (\underline{|Q_s|} + 3)\right)$$

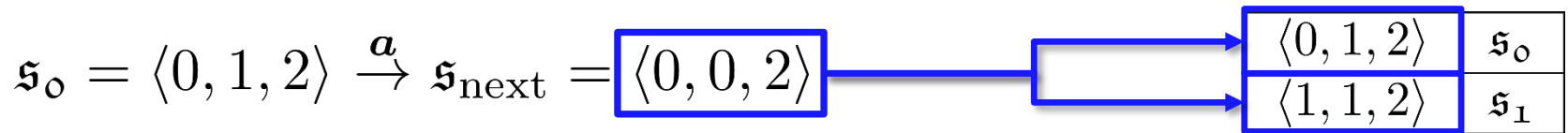
An SFA size  $|Q_s|$  is  $\underline{\mathcal{O}(|Q|^{|Q|})}$  in the worst case

**Exponential state-growth**

# Fingerprint-based hashing

## □ Fingerprints ( $F$ )

- ▣ Short bit-strings for larger objects (SFA-states)
- ▣ **CityHash**, FarmHash, Rabin's method, etc. create fingerprints
- ▣ Speed up comparisons of SFA-states

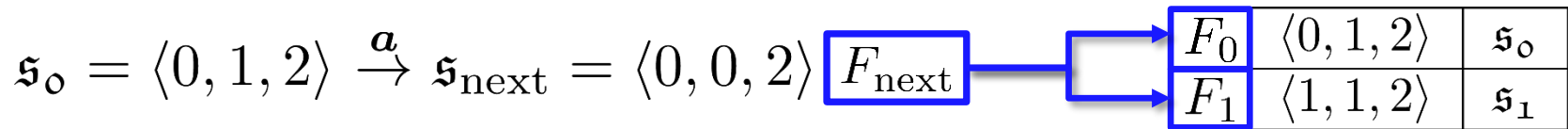


$\mathcal{O}(|Q|)$  exhaustive SFA-state comparisons

# Fingerprint-based hashing

## □ Fingerprints ( $F$ )

- ▣ Short bit-strings for larger objects (SFA-states)
- ▣ **CityHash**, FarmHash, Rabin's method, etc. create fingerprints
- ▣ Speed up comparisons of SFA-states

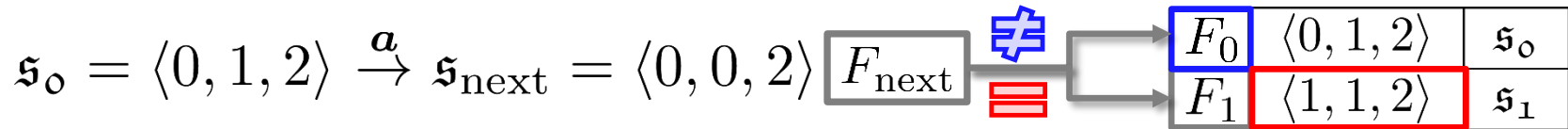


$\mathcal{O}(1)$  fingerprint comparisons

# Fingerprint-based hashing

## □ Fingerprints ( $F$ )

- Short bit-strings for larger objects (SFA-states)
- CityHash**, FarmHash, Rabin's method, etc. create fingerprints
- Speed up comparisons of SFA-states



## □ Fingerprint-collisions

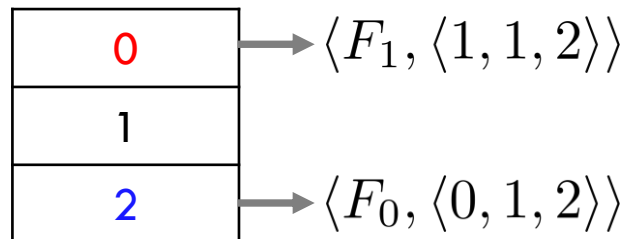
- It follows from the properties of the hash function that if fingerprints are **different**, SFA-states are **different**.
  - No exhaustive comparison necessary.
- With small probability, different SFA-states generate **same** fingerprint.
  - Fingerprint-collision
  - If fingerprints are the **same**, SFA-states **may** be the **same**.
    - $\mathcal{O}(|Q|)$  exhaustive comparisons are required.

# Fingerprint-based hashing (cont.)

## □ Hashing of SFA-states

- ▣ Speed up lookups, reduces number of SFA-state comparisons
- ▣ **Hash key:** fingerprint % size of the hash-table
- ▣ **Value:**  $\langle \text{fingerprint, SFA-state} \rangle$

$s_0$	$F_0 = 2$	$\langle 0, 1, 2 \rangle$
$s_1$	$F_1 = 0$	$\langle 1, 1, 2 \rangle$



**Hash-table** (size=3)

# Fingerprint-based hashing (cont.)

## □ Hash-collisions

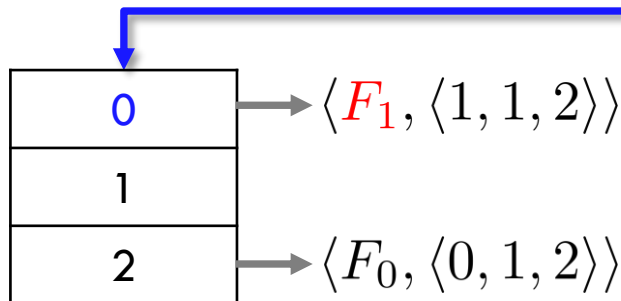
- ▣ Different SFA-states may map to the same hash-key due to the modulo-operation.

$\mathfrak{s}_0$	$F_0 = 2$	$\langle 0, 1, 2 \rangle$
$\mathfrak{s}_1$	$F_1 = 0$	$\langle 1, 1, 2 \rangle$

$$\mathfrak{s}_0 = \langle 0, 1, 2 \rangle \xrightarrow{a} \mathfrak{s}_2 = \langle 0, 0, 2 \rangle$$

$$\downarrow$$
$$F_2 = 3$$

Hash-collision



Hash-table (size=3)

# Fingerprint-based hashing (cont.)

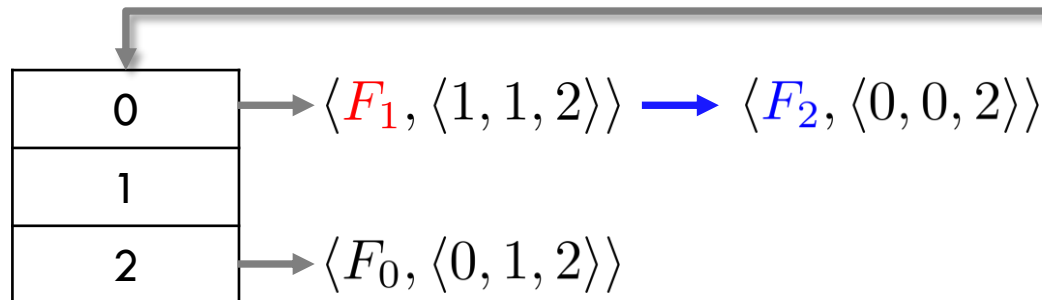
## □ Hash-collisions

- ▣ Different SFA-states may map to the same hash-key due to the modulo-operation.
- ▣ Resolved by closed addressing with chaining

$\mathfrak{s}_0$	$F_0 = 2$	$\langle 0, 1, 2 \rangle$
$\mathfrak{s}_1$	$F_1 = 0$	$\langle 1, 1, 2 \rangle$

$$\mathfrak{s}_0 = \langle 0, 1, 2 \rangle \xrightarrow{a} \mathfrak{s}_2 = \langle 0, 0, 2 \rangle$$

$$\downarrow$$
$$F_2 = 3$$



**Hash-table (size=3)**

# Parameterized transposition

- Speed up creating next SFA-states of each SFA-state

	a	b	c
0	1	0	0
1	1	0	2
2	2	2	2

DFA transition table

$$s_0 = \langle 0, 1, 2 \rangle \xrightarrow{a} s_1 = \langle 1, 1, 2 \rangle$$

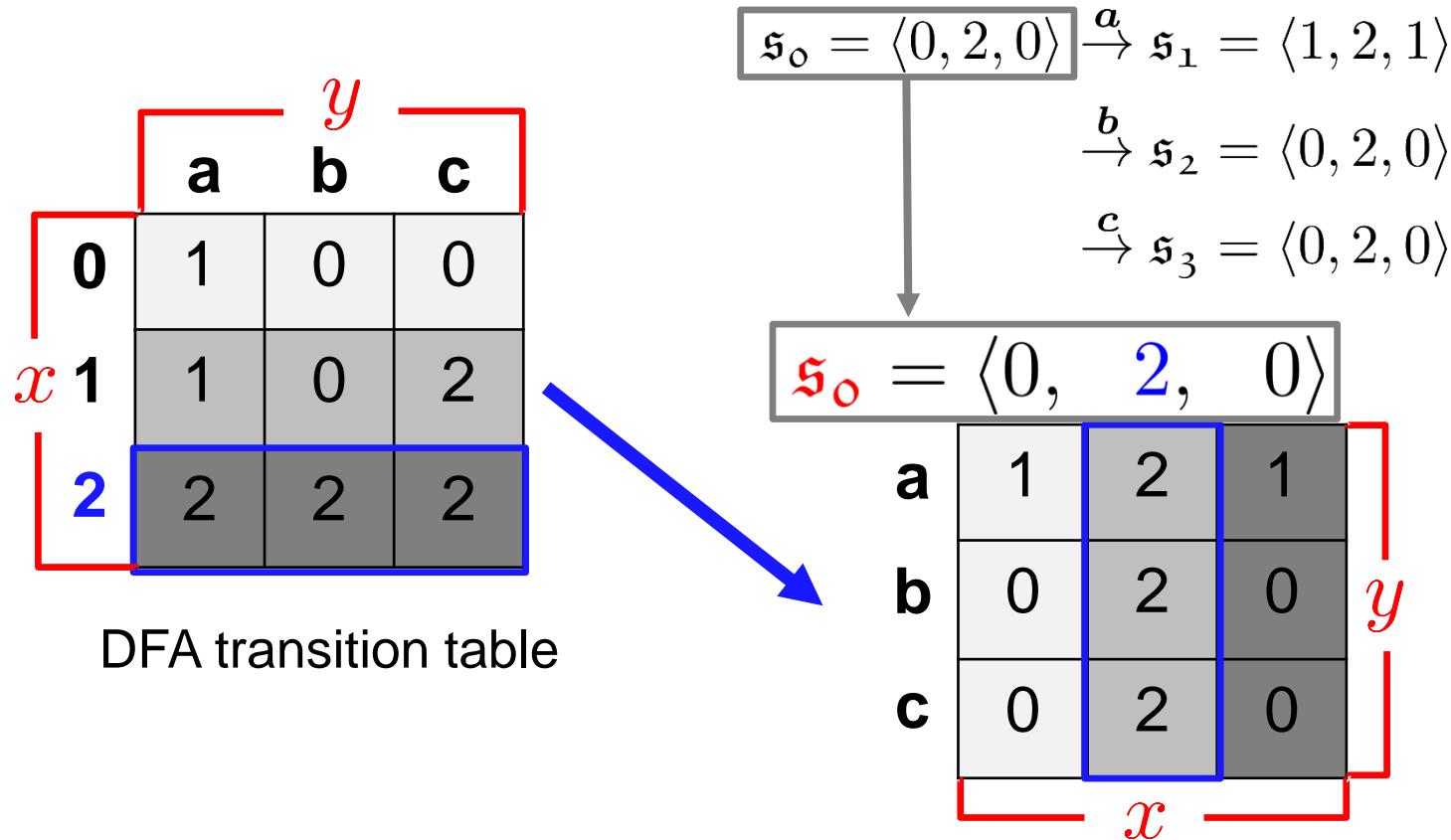
$$\xrightarrow{b} s_2 = \langle 0, 0, 2 \rangle$$

$$\xrightarrow{c} s_3 = \langle 0, 2, 2 \rangle$$

**Non-optimized:**  
compute next states one by one

# Parameterized transposition

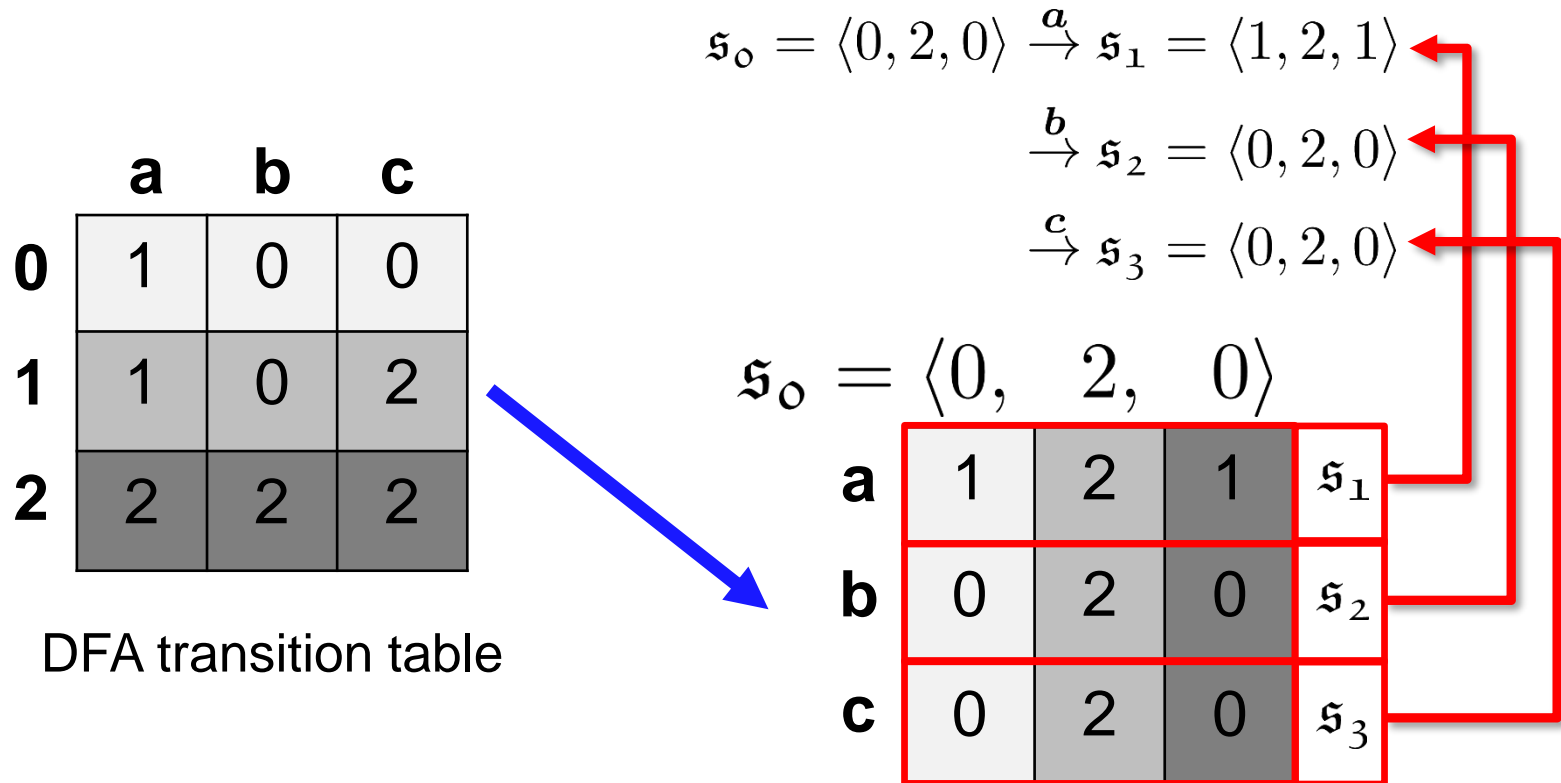
- Speed up creating next SFA-states of each SFA-state



**Optimized:** transpose the  $x \times y$  table to the  $y \times x$  table according to the DFA-states of the **source** SFA-state

# Parameterized transposition

- Speed up creating next SFA-states of each SFA-state

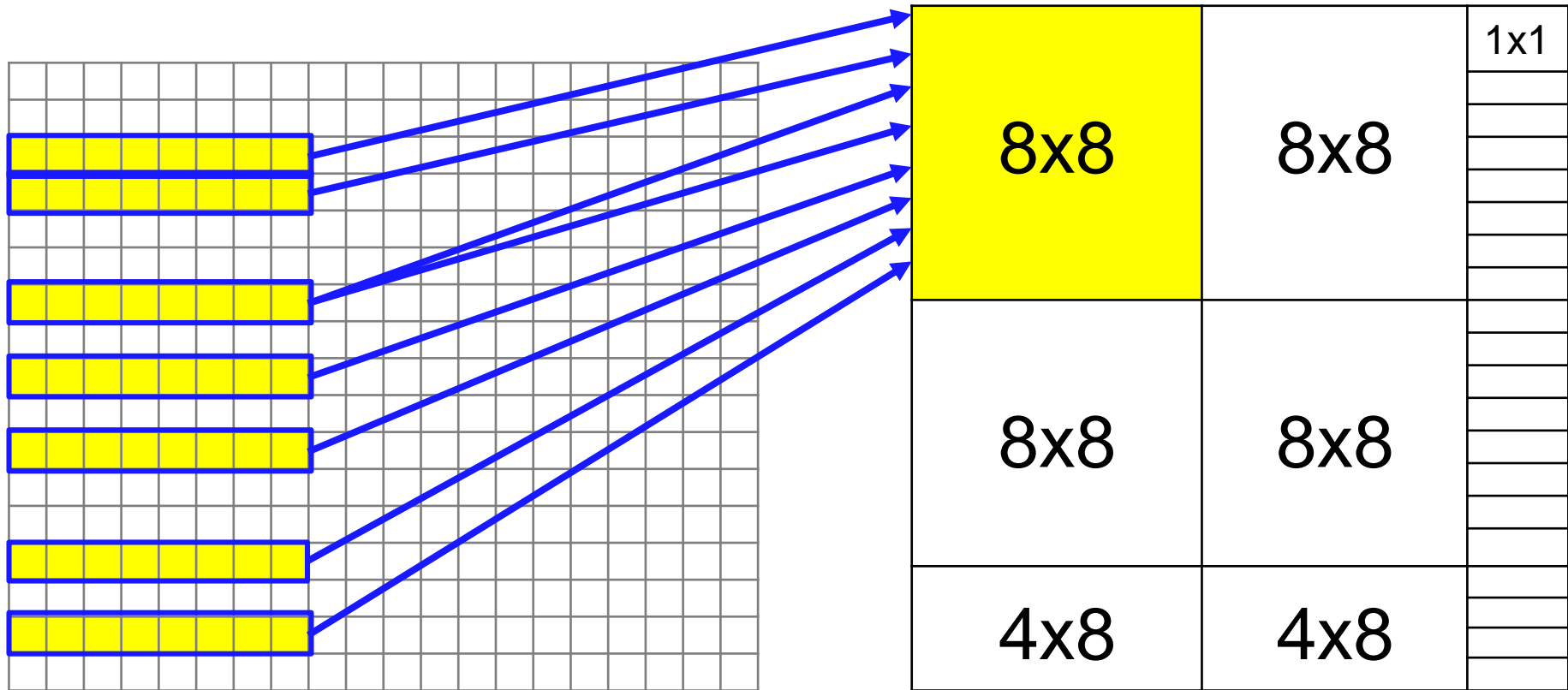


**Optimized:** transpose the  $x \times y$  table to the  $y \times x$  table according to the DFA-states of the **source** SFA-state

# Parameterized transposition (cont.)

## □ Example transposed transition table

▣ # DFA-states: 17, # symbols: 20



DFA transition table (17x20)



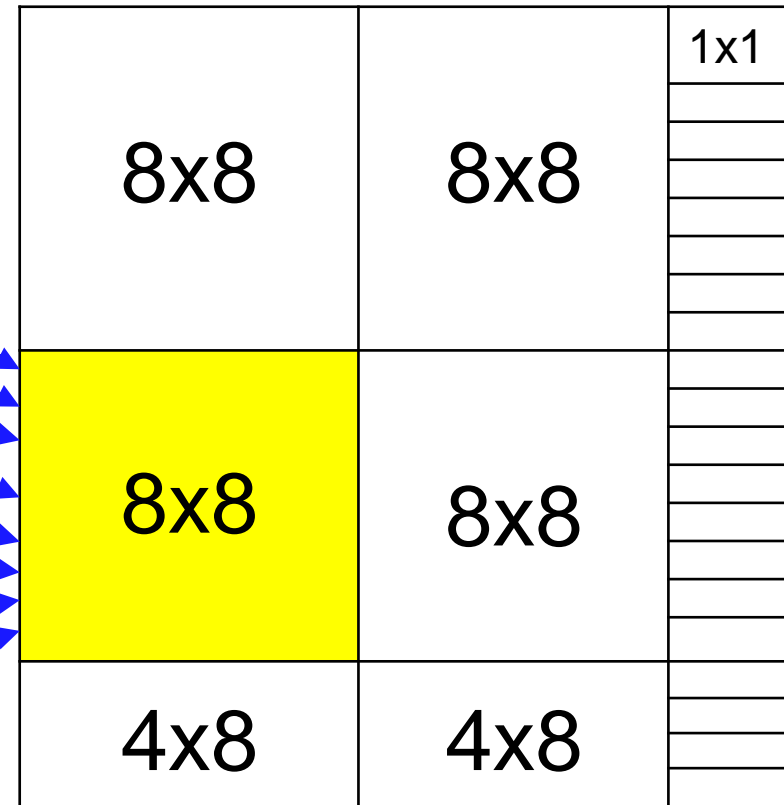
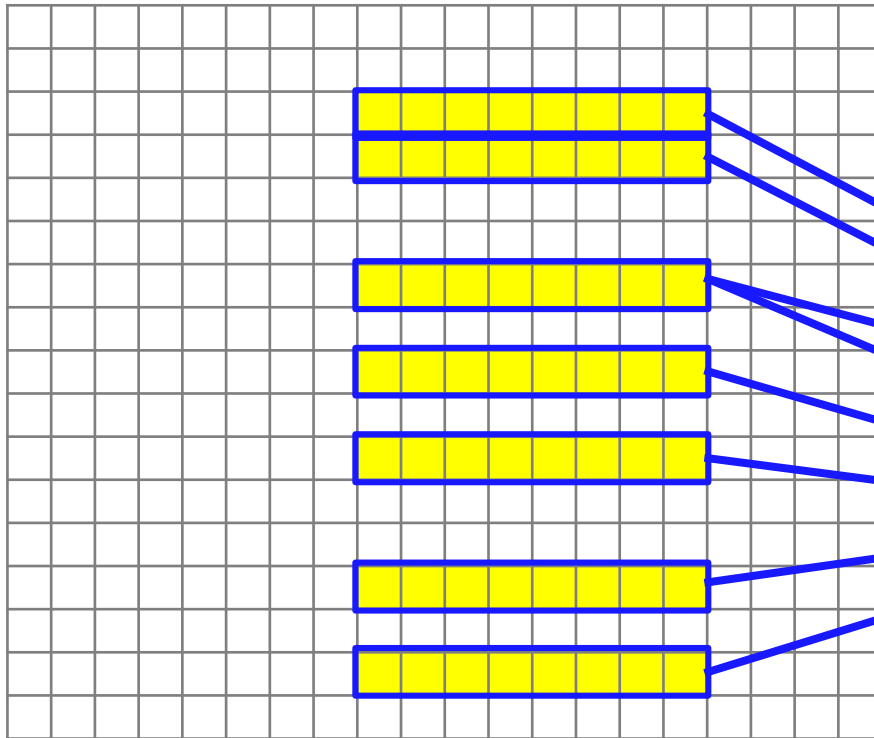
20 next SFA-states (20x17)

**x86 SIMD-intrinsics-based transposition kernels**

# Parameterized transposition (cont.)

## □ Example transposed transition table

▣ # DFA-states: 17, # symbols: 20



DFA transition table (17x20)



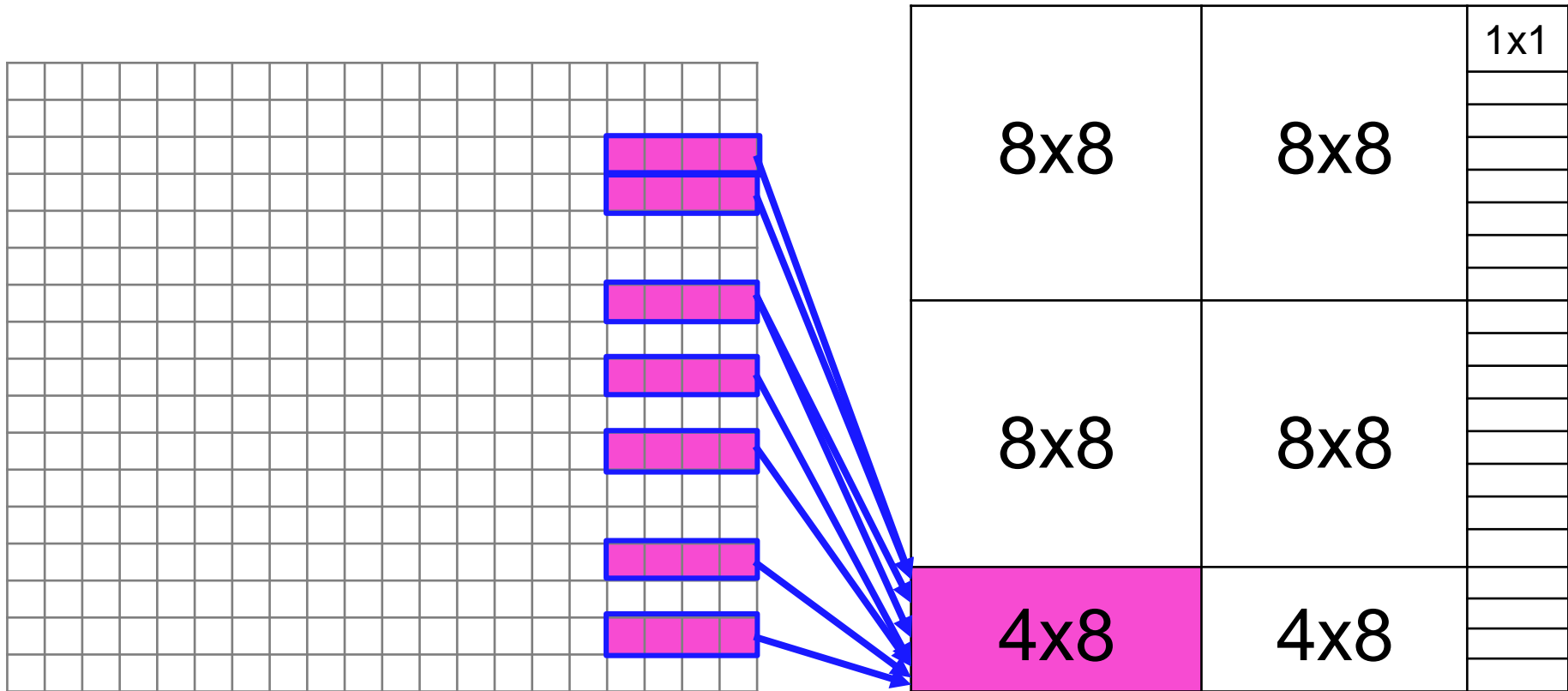
20 next SFA-states (20x17)

**x86 SIMD-intrinsics-based transposition kernels**

# Parameterized transposition (cont.)

## □ Example transposed transition table

▣ # DFA-states: 17, # symbols: 20



DFA transition table (17x20)

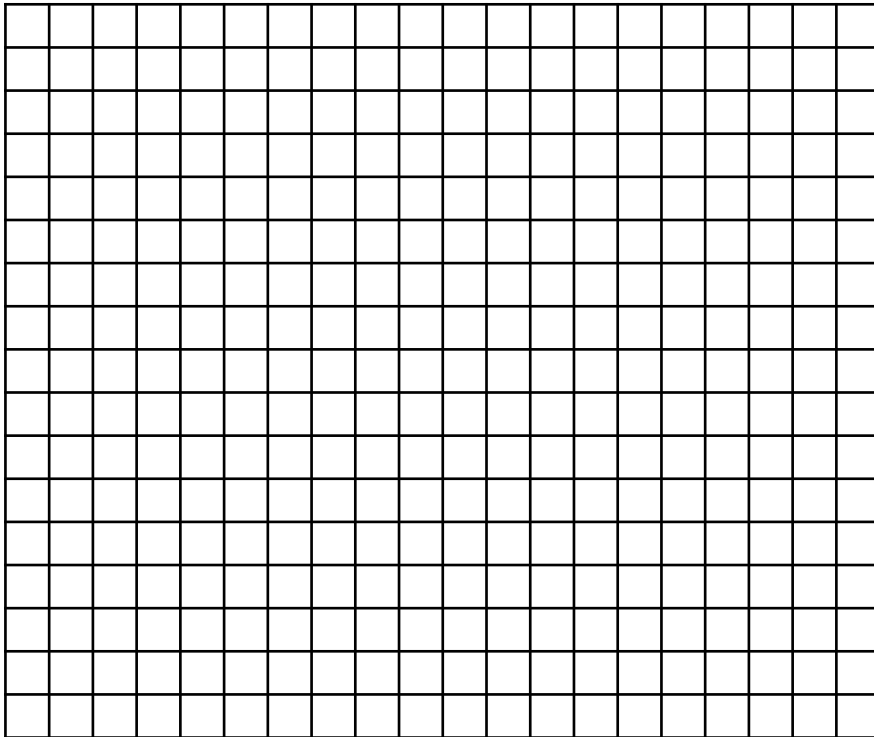
20 next SFA-states (20x17)

**x86 SIMD-intrinsics-based transposition kernels**

# Parameterized transposition (cont.)

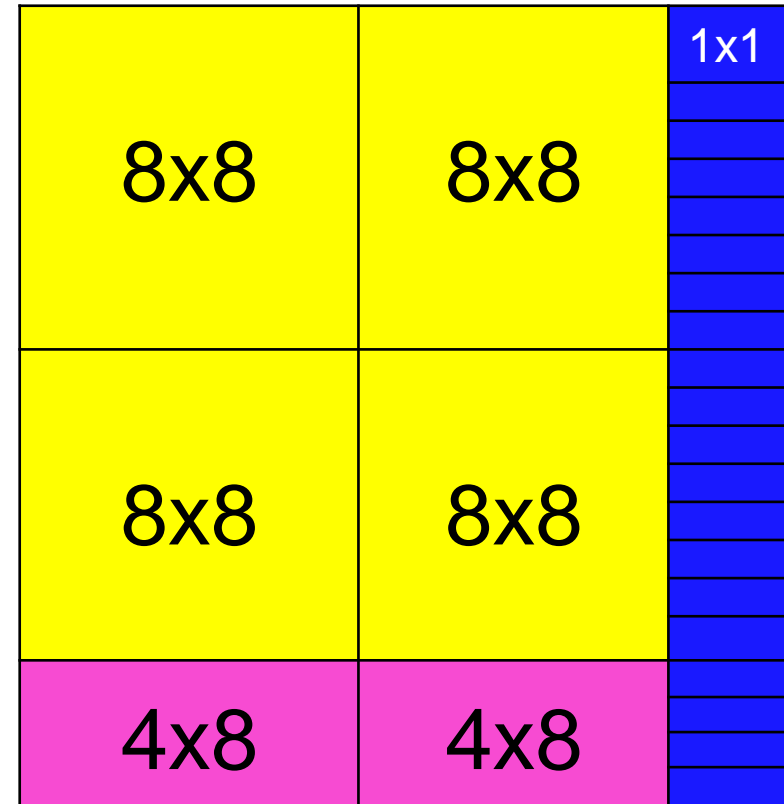
## □ Example transposed transition table

▣ # DFA-states: 17, # symbols: 20



A 17x20 grid representing the DFA transition table. The grid is composed of 17 rows and 20 columns of small squares.

DFA transition table (17x20)



A 20x17 grid representing the transposed transition table. The grid is partitioned into four 8x8 blocks (yellow) and two 4x8 blocks (pink) in the bottom row, and a 1x1 block (blue) in the top row. The blocks are labeled with their dimensions: 8x8, 8x8, 8x8, 8x8, 4x8, 4x8, and 1x1.

20 next SFA-states (20x17)

**x86 SIMD-intrinsics-based transposition kernels**

# Work (SFA-state) distribution

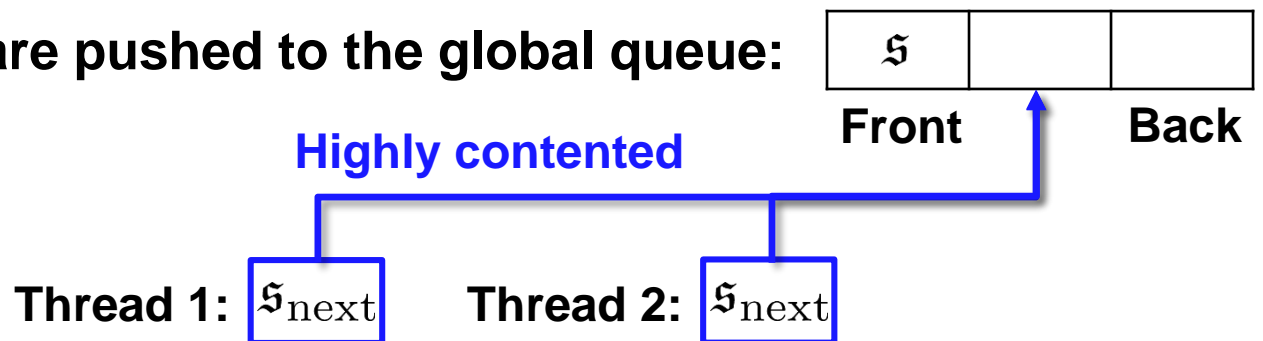
## □ Observations:

- 1) The amount of work changes dynamically.
  - Few available states at the beginning, but soon all cores are saturated.
- 2) Switching the work distribution scheme dynamically adapts to the changing load condition and reduces the cache-coherence overhead.

## □ **Scheme 1:** static distribution via a global queue:

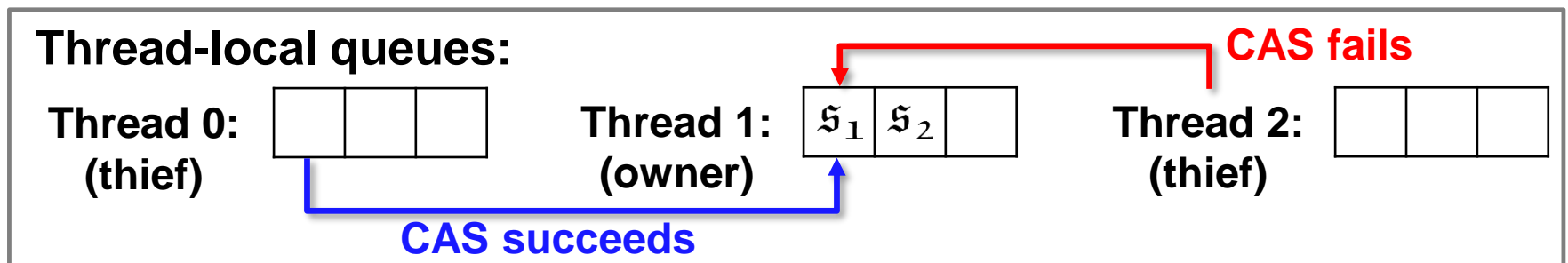
- **Advantage:** avoid coherence-overhead at front of the queue from work-stealing attempts of idle threads
- Back of the queue is not contended because initially little work is available.

**New SFA-states are pushed to the global queue:**



# Work (SFA-state) distribution (cont.)

- **Scheme 2:** dynamic distribution via thread-local queues
  - **Work-stealing:** steal work from the other's queue once the local queue is empty
    - Work will be popped exactly once by a thread because of lock-free synchronization using compare-and-swap (CAS) operation
  - **Advantage:** avoid coherence-overhead from the highly contended back of the global queue
  - Dequeueing SFA-states from other thread-local queues (work-stealing) makes front of the queue highly contended (cache coherence overhead) when little work is available



# In-memory compression

- SFA-state compression mitigates **state explosion** problem

$$\mathfrak{s}_0 = \langle 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, \dots, |Q| - 1 \rangle$$

27 KB per SFA-state

Compress

- Dictionary-based compression shows **high compression ratios** due to structural properties of FAs

- FA-states tend to repeat in SFA-states

$$\mathfrak{s}_1 = \langle 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 10, 0, 0, 0, 0, \dots, |Q| - 1 \rangle$$

- Compression requires additional **costly** computation
- Initiate once a critical memory threshold is reached

# In-memory compression (cont.)

- Mitigate **intractable** problem sizes
- Conduct SFA construction in three phases
  - ▣ **First phase**: construct an SFA with un-compressed SFA-states

$$\begin{aligned} \mathfrak{s}_0 = \langle 0, 1, 2 \rangle &\xrightarrow{b} \mathfrak{s}_1 = \langle 1, 1, 2 \rangle \\ &\xrightarrow{a} \mathfrak{s}_2 = \langle 0, 0, 2 \rangle \\ &\xrightarrow{c} \mathfrak{s}_3 = \langle 0, 2, 2 \rangle \end{aligned}$$

$\mathfrak{s}_0$	$\langle 0, 1, 2 \rangle$
$\mathfrak{s}_1$	$\langle 1, 1, 2 \rangle$
$\mathfrak{s}_2$	$\langle 0, 0, 2 \rangle$
$\mathfrak{s}_3$	$\langle 0, 2, 2 \rangle$

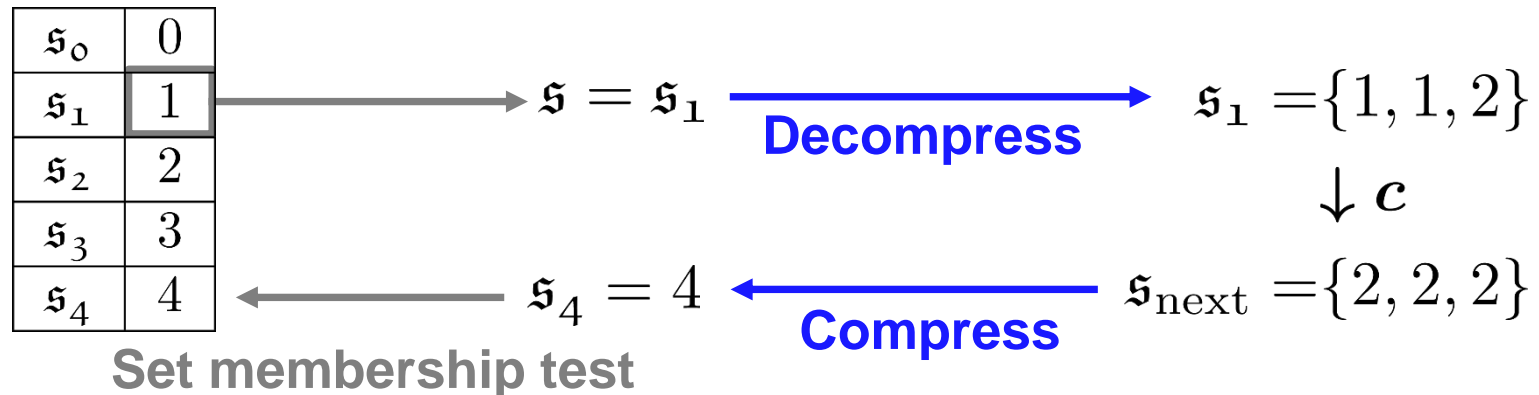
# In-memory compression (cont.)

- Mitigate **intractable** problem sizes
- Conduct SFA construction in three phases
  - ▣ First phase: construct an SFA with un-compressed SFA-states
  - ▣ **Second phase**: compress all generated SFA-states once a critical memory threshold is reached

$s_0$	$\langle 0, 1, 2 \rangle$	<b>Dictionary-based lossless compression</b> →	$s_0$	0
$s_1$	$\langle 1, 1, 2 \rangle$		$s_1$	1
$s_2$	$\langle 0, 0, 2 \rangle$		$s_2$	2
$s_3$	$\langle 0, 2, 2 \rangle$		$s_3$	3

# In-memory compression (cont.)

- Mitigate **intractable** problem sizes
- Conduct SFA construction in three phases
  - ▣ First phase: construct an SFA with un-compressed SFA-states
  - ▣ Second phase: compress all generated SFA-states once a critical memory threshold is reached
  - ▣ **Third phase**: resume SFA construction with compressed SFA-states

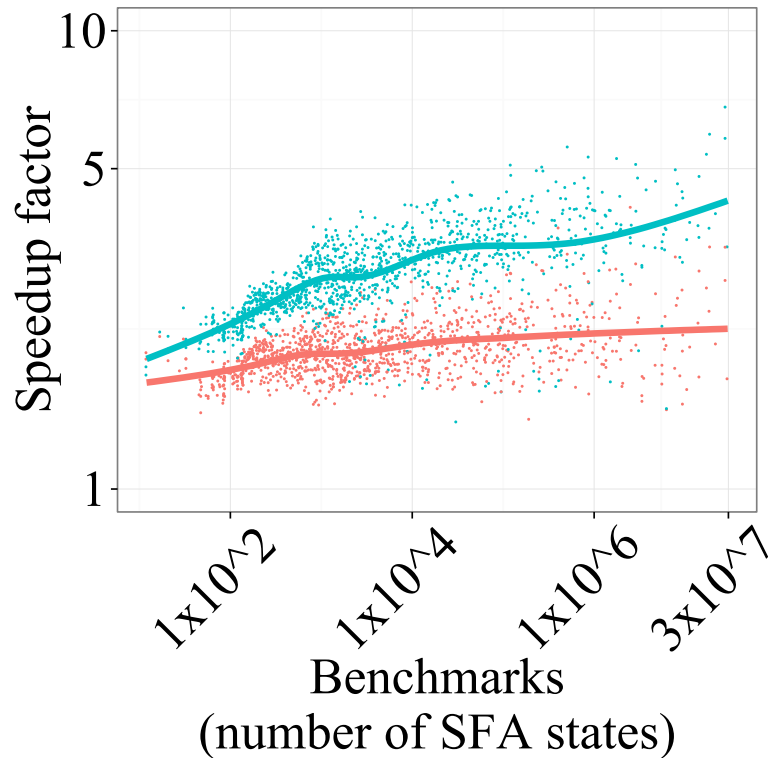


# Experimental evaluation

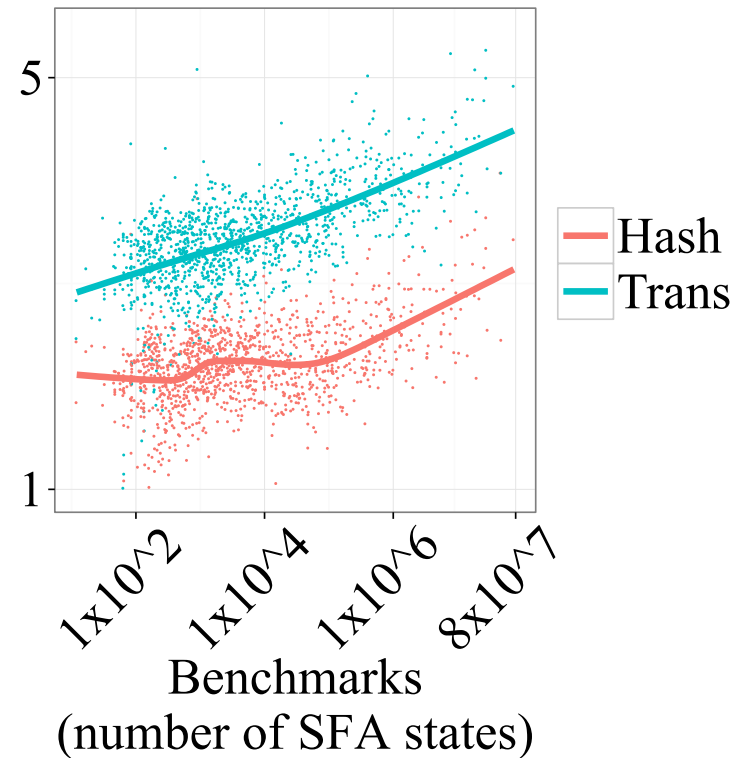
- ❑ Benchmarks: 1250 patterns from PROSITE protein database
  - ❑ Their minimal DFAs are generated by Grail+.
  - ❑ Exclude patterns take several days to convert to minimal DFAs.
- ❑ Proposed algorithm implemented in C11 using POSIX threads.
- ❑ Performance results are obtained by PAPI allows accessing hardware performance counters.
- ❑ Evaluation platforms:
  - ❑ 4-CPU (64 cores) AMD Opteron system
  - ❑ 2-CPU (44 cores, 2 hyperthreads per core) Intel Xeon Broadwell E5-2699 v4 system
  - ❑ Linux CentOS version 7

# Experimental evaluation (cont.)

- Speedups of optimized sequential algorithm over the previous algorithm



**On the AMD system**



**On the Intel system**

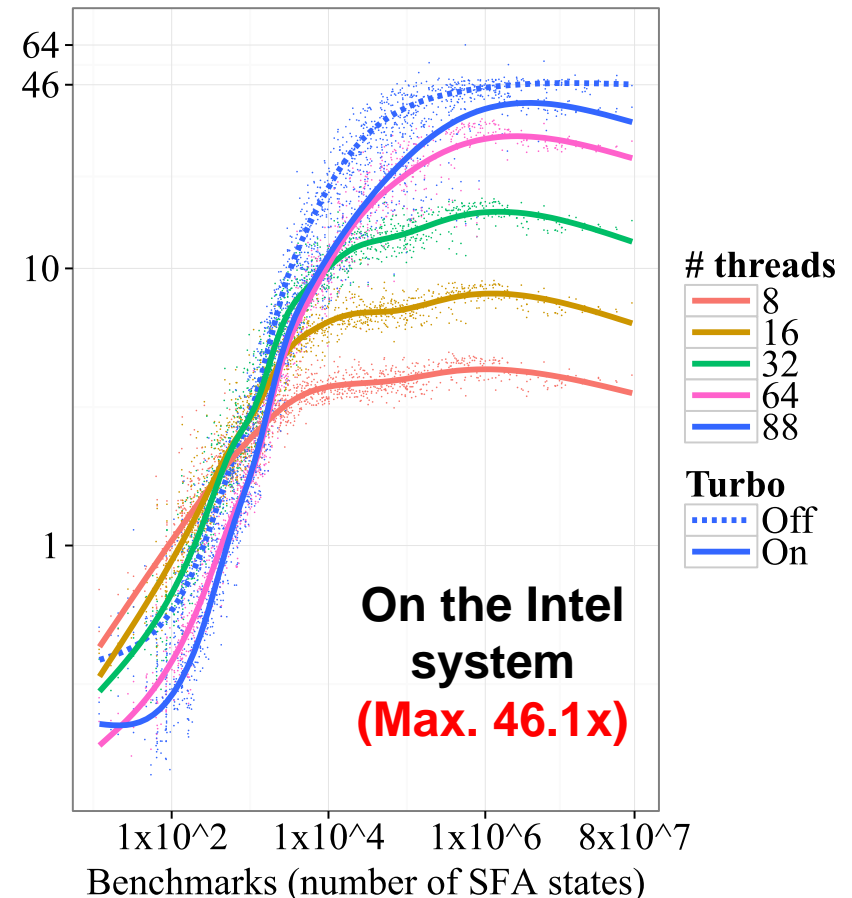
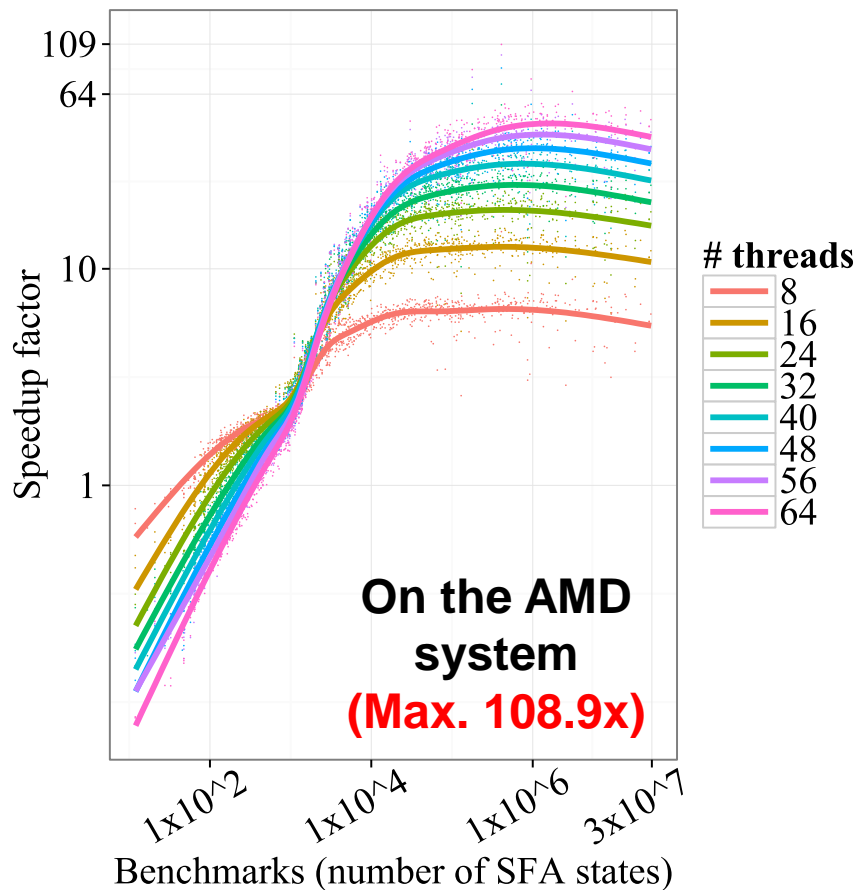
□ **Hashing**: max 4.1x on AMD, 3.1x on Intel

□ **Combination of hashing and transposition**:

max 6.8x on AMD, 5.2x on Intel

# Experimental evaluation (cont.)

- Speedups of parallelization
  - ▣ Based on our fastest sequential algorithm using hashing and parameterized transposition



# Experimental evaluation (cont.)

- Performance and size comparison with and w/o compression
  - ▣ Six benchmarks on the Intel system (four benchmarks are intractable w/o compression and two benchmarks are added to compare them)
  - ▣ Set our memory manager's threshold to 200 GB to force compression of two tractable benchmarks

Number of States		w/o compr.		with compr.		Compr. Ratio
DFA	SFA	Size (MB)	Time (s)	Size (MB)	Time (s)	
2,557	74,624,878	381,632	42	12,622	1,015	30
2,980	40,956,096	244,098	28	13,172	436	19
6,132	17,795,082	436,478	n/a	12,153	824	18
6,419	20,559,280	527,880	n/a	15,495	1,212	17
6,549	47,076,417	1,233,214	n/a	29,610	2,700	21
7,025	23,975,400	673,709	n/a	20,106	1,641	17



**Intractable w/o compression**

# Conclusion

- ❑ Introduced **fingerprints** and **hashing** to reduce state comparisons and set membership tests.
- ❑ **Parameterized transposition** of the transition table ensures cache locality of memory accesses.
- ❑ **Dynamic switch** from global work queue to **thread local queues** with work-stealing avoids contention of cache-lines at front and back of queue.
- ❑ Dynamically switch to **in-memory compression** of SFA-states once they cannot fit into the main memory.
- ❑ **Overall speedups** including fingerprint-based hashing, parameterized transposition and parallelization without compression are up to **312x** on AMD and **193x** on Intel.
- ❑ **Compression ratios** are up to **30** on the Intel system.

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  - ▣ the Austrian Science Fund (FWF) project I 1035N23
  - ▣ the Next-Generation Information Computing Development Program through the National Research Foundation of Korea (NRF), funded by the Ministry of Science, ICT & Future Planning under grant NRF2015M3C4A7065522

Thank you!